

Homework Problem #027

Determine the average and effective values of the voltage signal

$$v(t) = 30 \cos(10t) + 40 \cos(20t) + 50 \cos(30t) \text{ V}$$

$$\begin{aligned} T_1 &= \frac{2\pi}{10} & T_2 &= \frac{2\pi}{20} & T_3 &= \frac{2\pi}{30} \\ &= \frac{\pi}{5} \text{ s} & &= \frac{\pi}{10} \text{ s} & &= \frac{\pi}{15} \text{ s} \end{aligned}$$

$$\text{overall } T = \text{LCM}\left(\frac{\pi}{5}, \frac{\pi}{10}, \frac{\pi}{15}\right) = \frac{\pi}{5} \text{ s}$$

$$\begin{aligned} V_{\text{ave}} &= \frac{1}{T} \int_0^T [30 \cos(10t) + 40 \cos(20t) + 50 \cos(30t)] dt \\ &= \frac{1}{T} \int_0^T 30 \cos(10t) dt + \frac{1}{T} \int_0^T 40 \cos(20t) dt + \frac{1}{T} \int_0^T 50 \cos(30t) dt \\ &= \frac{\pi}{5} \cdot 30 \cdot \int_0^{\pi/5} \cos(10t) dt + \frac{\pi}{5} \cdot 40 \cdot 2 \int_0^{\pi/10} \cos(20t) dt + \frac{\pi}{5} \cdot 50 \cdot 3 \int_0^{\pi/15} \cos(30t) dt \\ &= \frac{150}{\pi} (0) + \frac{400}{\pi} (0) + \frac{650}{\pi} (0) = 0 \text{ V} \end{aligned}$$

$$\begin{aligned} V_{\text{effective}}^2 &= \frac{1}{T} \int_0^T [30 \cos(10t) + 40 \cos(20t) + 50 \cos(30t)]^2 dt \\ &= \frac{1}{T} \int_0^T \left[900 \cos^2(10t) + 1200 \cos(10t) \cos(20t) \right. \\ &\quad + 1500 \cos(10t) \cos(30t) + 1200 \cos(10t) \cos(20t) \\ &\quad + 1600 \cos^2(20t) + 2000 \cos(20t) \cos(30t) \\ &\quad + 1500 \cos(10t) \cos(30t) + 2000 \cos(20t) \cos(30t) \\ &\quad \left. + 2500 \cos^2(30t) \right] dt \\ &= \frac{900}{2} + \frac{1600}{2} + \frac{2500}{2} = \frac{5000}{2} = 2500 \end{aligned}$$

$$V_{\text{eff}} = \sqrt{2500} = 50 \text{ V}$$